Beyond Finite Layer Neural Network: Bridging Deep Architects and Numerical Differential Equations

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Introduction

In this work, we show that many effective networks, such as ResNet, PolyNet, FractalNet and RevNet, can be interpreted as different numerical discretizations of differential equations。
We also propose a linear multi-step architecture (LM-architecture) which is inspired by the linear multi-step method solving ordinary differential equations. The LM-architecture is an effective structure that can be used on any ResNet-like networks。

Neural Network As Numerical Scheme

Our motivation is to consider the residual network $x_n = x_{n-1} + f(x_{n-1})$ as the forward Euler scheme for the ODE $\dot{X} = f(X)$. We found out that many neural network can be consider as different numerical scheme for ODE , some examples is listed below.

Table1. Neural Networks' associated ODES and numerical scheme

Network	Related ODE	Numerical Scheme
ResNet, ResNeXt	$u_t = f(u)$	Forward Euler
PolyNet	$u_t = f(u)$	Approximation of Backward Euler
Fractal Net	$u_t = f(u)$	Runge-Kutta
RevNet	$u_t = f(v), v_t = g(u)$	Forward Euler

LM(Linear Multistep)-ResNet

We utilize the linear multi-step scheme to discrete the ODE, then we get a new type of neural network:

$$x_n = (1 - k_n)x_{n-1} + k_n x_{n-2} + f(x_{n-1})$$

Our neural network only inject one parameter in each layer of a ResNet but shows a huge improvement.

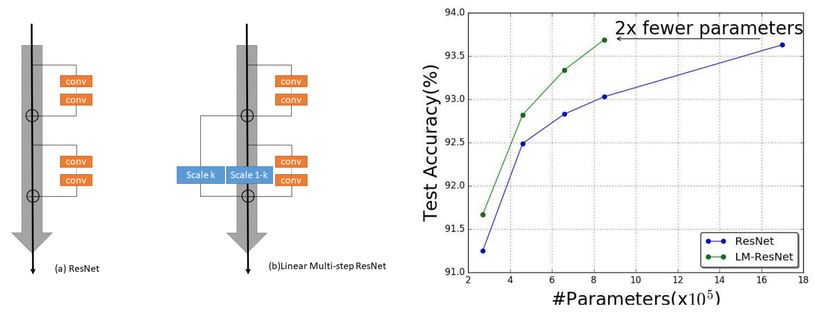


Fig1. Our LM-ResNet and numerical result on CIFAR10.

Benefit Of LM Structure

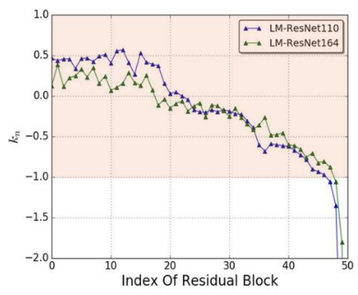
We analysis our scheme by modified equation, which is a new equation approximated by the scheme in a higher order.

ResNet:

$$\frac{\Delta t}{2}\ddot{X} + \dot{X} = f(u)$$

LM-ResNet:

$$(1 - k_n)\frac{\Delta t}{2}\ddot{X} + (1 + k_n)\dot{X} = f(u)$$



In short, we find out that the parameterk_n learned in the LM-ResNet introduced a momentum in to the dynamic of information propagating in network.

Table2. LM-ResNeXt On CIFAR100

Model	Layer	Top1	Top5
ResNeXt	29(8x64d)	17.77	34.4M
ResNeXt	29(16x64d)	17.31	68.1M
ResNeXt	29(16x64d), pre-act	17.65	68.1M
LM-ResNeXt	29(8x64d), pre-act	17.49	34.4M
LM-ResNeXt	29(16x64d), pre-act	16.79	68.1M

Table3. Single-crop Error Rate On Imagenet

Model	Layer	Top1	Top5
ResNet	50	24.7	7.8
ResNet	101	23.6	7.1
ResNet	153	23.0	6.7
LM-ResNet	50, pre-act	24.0	7.3
LM-ResNet	101. pre-act	22.6	6.4

Stochastic Learning Meets Stochastic Control

We also find some stochastic learning method can be consider as approximating a stochastic dynamic system.

Stochastic Depth

$$dX = p(t)f(X)dt + \sqrt{p(t)(1-p(t))}f(X) \odot [1_{N\times 1}, 0_{N\times 1}]dB_t$$

Shake-Shake

$$dX = \frac{1}{2}(f_1(X) + f_2(X))dt + \frac{1}{\sqrt{12}}(f_1(X) - f_2(X)) \odot [1_{N\times 1}, 0_{N\times 1}]dB_t$$

So that we can consider stochastic learning method as solving a general stochastic control problem.

$$\min \mathbb{E}_{X(0)\sim data}\left(\mathbb{E}\left(l(X(t))\right) + \int_0^t R(\theta)\right)$$
$$s.t. \partial_t X = f(X,\theta) + g(X,\theta)dB_t$$

We also tested stochastic learning version of LM-ResNet, which also boosted the performance.

Model	Layer	Training Strategy	Error
ResNet	110		6.61
ResNet	110,pre-act		6.37
ResNet	110	Stochastic Depth	5.25
ResNet	1202	Stochastic Depth	4.91
LM-ResNet	56,pre-act	Stochastic Depth	5.14
LM-ResNet	110,pre-act	Stochastic Depth	4.80

Table4. Result Of LM-ResNet With Stochastic Depth On CIFAR10

Reference

E, Weinan. A proposal on machine learning via dynamical systems. Communications in Mathematics and Statistics, 5(1):1–11, 2017.
He, Kaiming, Zhang, Xiangyu, Ren, Shaoqing, and Sun, Jian. Identity mappings in deep residual networks. IEEE Conference on Computer Vision and Pattern Recognition, 2016.